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The influence of Coriolis force on surface-tension-driven convection

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The effect of uniform rotation on surface-tension-driven convection in an evaporating fluid layer is considered both theoretically and experimentally. The theoretical analysis follows the usual small-disturbance approach of perturbation theory and leads, at the neutral state, to a functional relation between the Marangoni and Taylor numbers which is then computed numerically. In addition, it is shown analytically that, in the limit of rapid rotation, the velocity and temperature fluctuations are confined to a thin Ekman layer near the surface, and that $M_c = 4{\cdot}42T^{\frac{1}{2}}$ and $\alpha_c = 0{\cdot}5T^{\frac{1}{4}}$, where M_c and α_c are, respectively, the critical Marangoni number and the critical wave number for neutral stability, and T is the Taylor number.

The experimental part deals primarily with the flow pattern of a 50 % solution of ethyl ether in n-heptane evaporating into still air. In this case, the convective flow is surface-tension-driven and its structure was observed using schlieren optics. In the absence of rotation, the flow shows a remarkable cellular pattern when the layer is shallow, but when the depth of the layer is increased the pattern quickly becomes highly irregular. In contrast, for $T > 10^3$, a cellular structure is always observed even for deep layers, a result which is attributable to the stabilizing effect of the Coriolis force. A further increase in T leaves the flow pattern unchanged except that the size of the cells is found to decrease as $T^{-\frac{1}{4}}$, which is in agreement with the results of the linear stability analysis.

1. Introduction

As is well known, the convective currents arising spontaneously in evaporating liquids are generally due to two fairly dissimilar mechanisms: the familiar buoyancy mechanism, and the somewhat less familiar but equally important surface-tension mechanism which, as shown analytically by Pearson (1958), can cause instability in systems that would be classified as stable according to the Rayleigh criterion. The presence of such convective flows is often considered desirable in many operations involving heat or mass transfer because it increases the rate of transport of energy and matter. On the other hand, there exist a number of important cases involving the determination of certain physical properties such as the condensation coefficient or the interfacial resistance to mass transfer from a liquid to a gas phase, where it is imperative that these currents be suppressed to the fullest possible extent. Buoyancy-driven convection can be eliminated of course by operating with relatively shallow liquid layers, i.e. less than about 5 mm deep

(Berg, Boudart & Acrivos 1966). In contrast, surface-tension-driven convection seems to persist even in pools 0·5 mm in depth (Berg *et al.* 1966) which, from a practical point of view, represents the minimum thickness of the layer that one can operate with experimentally. Although it is true that one could eliminate this type of convection, especially for water, by using small amounts of surface active agents the stabilizing effects of which are quite pronounced (Berg & Acrivos 1965), such a technique would have an obvious disadvantage in that it would affect greatly the very nature of the surface and hence precisely those physical properties that one wishes to measure.

In view of the extraordinary stability of rotating fluids, it appears logical to suppose, however, that the three-dimensional convective motions in evaporating layers could be damped by subjecting the system to uniform rotation, a fact which has already been demonstrated by Chandrasekhar (1961) for the buoyancy-driven case. It is the purpose of this paper, therefore, to investigate theoretically the possible influence of such a Coriolis force on convection which is surface-tension driven, and to correlate the results of the mathematical analysis with those of an experimental study of convection in evaporating systems undergoing uniform rotation.

2. Linear stability analysis

Our mathematical treatment will follow the classical lines of linear stability theory as presented by Chandrasekhar (1961), whose notation we shall also adopt with only minor exceptions. We consider a system consisting of a liquid layer of infinite horizontal extent which is confined between two horizontal planes representing, respectively, a vapour-liquid interface at $z' = 0$ and an isothermal solid wall at $z' = d$. The basic equations are first linearized in terms of w', ζ' and θ' which denote, respectively, the perturbations in the z-component of the velocity, the z-component of the vorticity and the temperature, and are then rendered dimensionless by letting $(x, y, z) = (x'/d, y'/d, z'/d)$, $w = w'd/\nu$, $\zeta = \zeta' d^2/\nu$, $\theta = \theta'\kappa/\beta\nu d$, and $t = t'\nu/d^2$. Thus, we arrive at

$$\frac{\partial \zeta}{\partial t} - \nabla^2 \zeta = \surd T \frac{\partial w}{\partial z}, \tag{1}$$

$$\nabla^2 \left[\frac{\partial w}{\partial t} - \nabla^2 w\right] = -\surd T \frac{\partial \zeta}{\partial z}, \tag{2}$$

$$Pr(\partial\theta/\partial t) - \nabla^2\theta = -w, \tag{3}$$

where $Pr \equiv \nu/\kappa$ is the Prandtl number and $T \equiv 4\Omega^2 d^4/\nu^2$ the Taylor number. Here Ω denotes the constant speed of rotation, β the undisturbed temperature gradient across the layer, ν the kinematic viscosity of the liquid, and κ its thermal diffusivity. The equations shown above are of course identical to those in Chandrasekhar (1961) except for the absence of the buoyancy term. It is also clear that these equations apply even when the instability of the system is caused by a concentration rather than a temperature gradient, apart from the simple change of parameters

$$(\kappa, \beta, \theta, Pr) \rightarrow (\mathscr{D}, \alpha, c, Sc),$$

where $\mathscr{D}$ is the molecular diffusivity, α the steady-state concentration gradient, c the concentration disturbance, and $Sc \equiv \nu/\mathscr{D}$ the Schmidt number.

The boundary conditions at the isothermal solid surface $z = 1$ are

$$w = \partial w/\partial z = \zeta = \theta = 0, \tag{4}$$

while, at the vapour-liquid interface $z = 0$, considered non-deformable,*

$$w = \partial\zeta/\partial z = 0, \quad (\partial\theta/\partial z) + Nu\,\theta = 0, \quad (\partial^2 w/\partial z^2) + M\nabla_1^2\theta = 0, \tag{5}$$

where, as shown by Pearson (1958), the last relation, with $\nabla_1^2 \equiv \partial^2/\partial x^2 + \partial^2/\partial y^2$, represents the stress balance at the free surface. Here, $Nu \equiv qd/k$ denotes the vapour-phase Nusselt number for heat transfer and $M \equiv s\beta d^2/\mu\kappa$ the 'Marangoni' number (Scriven & Sternling 1960), where q is a vapour-phase heat-transfer coefficient, k is the thermal conductivity of the liquid, μ is its viscosity, and s represents the absolute value of the rate of change of the surface tension with temperature evaluated at the surface temperature.

The equations are now simplified in the usual manner by decomposing the solution in terms of normal modes, so that

$$\{w, \zeta, \theta\} \equiv \{f(z), h(z), g(z)\}\, F(x,y)\, e^{\sigma t}, \tag{6}$$

where σ is the dimensionless time constant, in general complex, and F satisfies the relation

$$\nabla_1^2 F = -\alpha^2 F$$

in which the 'wave-number' α, arising from the separation of variables, is indicative of the size of the transverse structure of the perturbation. Under the assumption that the neutral state is a stationary one, in which case $\sigma = 0$, the substitution of (6) into (1)–(3) leads, after elimination of the vorticity by cross-differentiation, to

$$[(D^2-\alpha^2)^3 + TD^2]f = 0 \quad \text{and} \quad (D^2-\alpha^2)g = f \tag{7}$$

with boundary conditions

$$\left.\begin{aligned} &f = Df = D(D^2-\alpha^2)^2 f = g = 0 \quad \text{at} \quad z = 1 \\ \text{and} \quad &f = (D^2-\alpha^2)^2 f = 0; \quad Dg + Nu\,g = 0, \quad D^2 f = \alpha^2 M g \quad \text{at} \quad z = 0, \end{aligned}\right\} \tag{8}$$

where $D \equiv d/dz$. Of course, this system reduces to that considered by Pearson when $T = 0$.

The general solution for f is clearly

$$f(z) = \sum_{j=1}^{6} A_j e^{\beta_j z}, \tag{9}$$

where the β_j's are the roots of

$$(\beta_j^2 - \alpha^2)^3 + T\beta_j^2 = 0,$$

while that for g is

$$g(z) = B_1 e^{\alpha z} + B_2 e^{-\alpha z} + \sum_{j=1}^{6} \frac{A_j}{\beta_j^2 - \alpha^2} e^{\beta_j z}. \tag{10}$$

* Scriven & Sternling (1964) have extended Pearson's (1958) stability analysis of surface-tension-driven convection by including the effect of surface deformation, which was shown to exert a destabilizing influence, especially for disturbances of very large wavelength. In the present treatment this extra complication will not be taken into account.

These, when substituted into the boundary conditions (8) yield a system of eight linearly independent linear homogeneous algebraic equations in A_j and B_j. The requirement that the determinant of the coefficients be zero in order to insure a non-trivial solution, provides us then with a characteristic

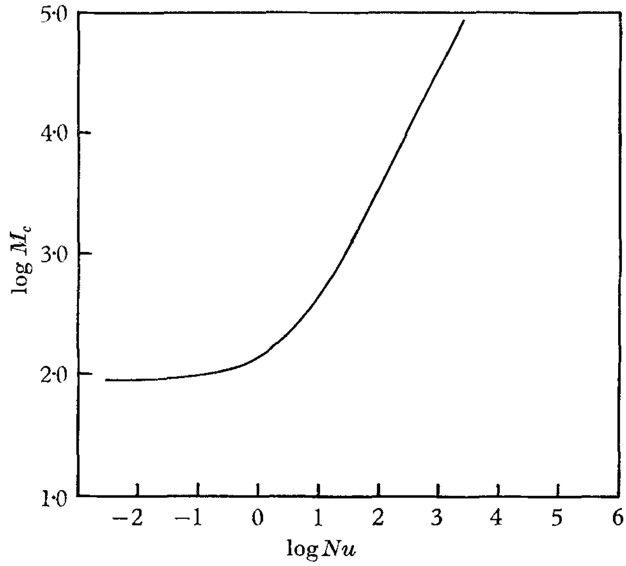

FIGURE 1. Critical value of the Marangoni number M_c vs. the vapour-phase Nusselt number Nu for $T = 100$.

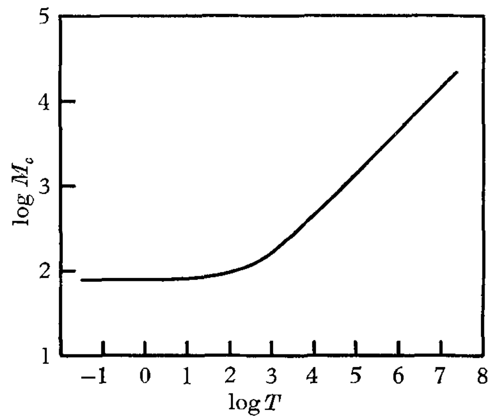

FIGURE 2. Critical value of the Marangoni number M_c vs. Taylor number T for $Nu = 0$.

equation relating the Taylor number T, the vapour-phase Nusselt number Nu, the wave-number α and the Marangoni number M. From this one can obtain finally the critical Marangoni number M_c, which refers to the minimum value of M for given T and Nu, as well as the corresponding critical wave-number α_c.

The effect of Nu on the stability of the system is shown in figure 1, which was constructed from a numerical solution of the characteristic equation. The system was found to be less stable when $Nu = 0$, which is as expected since clearly an isothermal free surface ($Nu \to \infty$) yields a completely stable configuration as far as surface-tension-driven convection is concerned.

The variation of the critical Marangoni number and the critical wave-number with T is depicted in figures 2 and 3 for the most unstable case, i.e. for $Nu = 0$. It is evident that both M_c and α_c increase monotonically with T and that, apparently,

$$M_c \sim T^{\frac{1}{2}} \quad \text{and} \quad \alpha_c \sim T^{\frac{1}{4}}$$

as $T \to \infty$. This last result we shall now establish by means of an asymptotic analysis of (7) and (8).

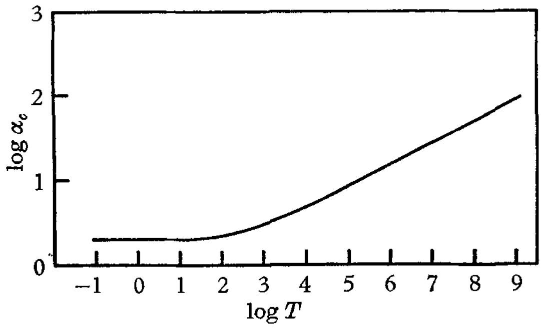

FIGURE 3. Critical value of the wave-number α_c vs. Taylor number T for $Nu = 0$.

3. Asymptotic solution for large Taylor numbers

It is instructive at this point to develop the asymptotic solution of (7) and (8) as $T \to \infty$. This is a straightforward matter because a simple order of magnitude analysis of the stability equations leads to the conclusion that $f = g = 0$ for all z except for a thin layer near the free surface, $z = 0$, where all the temperature and velocity fluctuations are confined. Hence, we seek a transformation of variables which will eliminate T from the system of equations in such a way that throughout this Ekman boundary layer all the principal terms in (7) and (8) will be retained. The appropriate transformations are then,

$$\mathscr{z} \equiv zT^{\frac{1}{4}}, \quad \alpha_0 \equiv \alpha T^{-\frac{1}{4}}, \quad G \equiv gT^{\frac{1}{4}}, \quad M_0 \equiv MT^{-\frac{1}{2}}, \quad D_0 \equiv d/d\mathscr{z},$$

following which (7) become

$$[(D_0^2 - \alpha_0^2)^3 + D_0^2]f = 0 \quad \text{and} \quad (D_0^2 - \alpha_0^2)G = f, \tag{11}$$

with boundary conditions

$$f = (D_0^2 - \alpha_0^2)^2 f = 0; \quad D_0^2 f = \alpha_0^2 M_0 G; \quad D_0 G = (Nu/T^{\frac{1}{4}})G \to 0 \quad \text{at} \quad \mathscr{z} = 0$$

together with the requirement that both f and G approach zero exponentially as $\mathscr{z} \to \infty$. We can see then that the transformed equations do indeed become independent of the Taylor number as $T \to \infty$ thus leading to a proper asymptotic solution of the original stability equations.

To solve the transformed system is now a simple matter. To begin with

$$f = \sum_{j=1}^{6} A_j \exp\{\beta_j \mathscr{z}\}, \tag{12}$$

where $$(\beta_j^2-\alpha_0^2)^3+\beta_j^2 = 0,$$

which becomes $$\gamma^3+\gamma+\alpha_0^2 = 0$$

upon setting $\gamma \equiv \beta_j^2-\alpha_0^2$. This expression has roots

$$\gamma_1 = A+B; \quad \gamma_2 = \overline{\gamma}_3 = -\tfrac{1}{2}(A+B)+\tfrac{1}{2}i\sqrt{3}(A-B),$$

where $$A = [-\tfrac{1}{2}\alpha_0^2+(\tfrac{1}{4}\alpha_0^4+\tfrac{1}{27})^{\frac{1}{2}}]^{\frac{1}{3}} \quad \text{and} \quad B = -[\tfrac{1}{2}\alpha_0^2+(\tfrac{1}{4}\alpha_0^4+\tfrac{1}{27})^{\frac{1}{2}}]^{\frac{1}{3}},$$

so that $$\beta_j = -\beta_{j+3} = \sqrt{(\gamma_j+\alpha_0^2)} \quad (j = 1,2,3).$$

Since, however, only β_1, β_2 and β_3 possess negative real parts, (12) simplifies to

$$f = \sum_{j=1}^{3} A_j \exp\{\beta_j z\},$$

where $$\beta_1 = -\sqrt{(\alpha_0^2-2a)}; \quad \beta_2 = \overline{\beta}_3 = -\sqrt{(\alpha_0^2+a+ib)},$$

with $a = -\frac{1}{2}(A+B)$ and $b = \frac{1}{2}\sqrt{3}(A-B)$. Also, the solution for G satisfying the condition $D_0 G = 0$ at $z = 0$ and $G \to 0$ as $z \to \infty$ is

$$G = -\frac{\cosh\alpha_0 z}{\alpha_0}\int_0^\infty e^{-\alpha_0 x} f(x)\,dx + \frac{1}{\alpha_0}\int_0^z \sinh\alpha_0(z-x) f(x)\,dx,$$

giving for G at $z = 0$

$$G(0) = -\frac{1}{\alpha_0}\int_0^\infty e^{-\alpha_0 x} f(x)\,dx = -\frac{1}{\alpha_0}\sum_{j=1}^{3}\frac{A_j}{\alpha_0-\beta_j}.$$

Hence, the remaining boundary conditions at $z = 0$ yield the three algebraic equations

$$\sum_{j=1}^{3} A_j = 0; \quad \sum_{j=1}^{3}\gamma_j^2 A_j = 0; \quad \sum_{j=1}^{3} A_j\left\{\beta_j^2+\frac{\alpha_0 M_0}{\alpha_0-\beta_j}\right\} = 0$$

from which one can derive the explicit expression for M_0,

$$M_0 = \frac{9a^2+b^2}{2\alpha_0 a} \times \left\{\frac{[\alpha_0+\sqrt{(\alpha_0^2-2a)}]\,[(\alpha_0+r^{\frac{1}{2}}\cos\frac{1}{2}\phi)^2+r\sin^2\frac{1}{2}\phi]}{(\alpha_0+r^{\frac{1}{2}}\cos\frac{1}{2}\phi)^2+r\sin^2\frac{1}{2}\phi-(\alpha_0+\sqrt{(\alpha_0^2-2a)})\,[\alpha_0+r^{\frac{1}{2}}\cos\frac{1}{2}\phi-2Rr^{\frac{1}{2}}\sin\frac{1}{2}\phi]}\right\}, \tag{13}$$

with $r \equiv \sqrt{\{(\alpha_0^2+a)^2+b^2\}}$, $\phi \equiv \tan^{-1} b/(\alpha_0^2+a)$ and $R \equiv (3a^2+b^2)/4ab$.

The dependence of M_0 on α_0, as obtained numerically from (13), is shown in figure 4. In particular, since a minimum value for M_0, equal to 4·42, was found at $\alpha_0 = 0{\cdot}50$,

$$M_c \to 4{\cdot}42 T^{\frac{1}{2}} \quad \text{and} \quad \alpha_c \to 0{\cdot}50 T^{\frac{1}{4}} \quad \text{as} \quad T \to \infty. \tag{14}$$

This result also holds asymptotically irrespective of the boundary conditions at $z = 1$.

For values of $T > 10^4$ these asymptotic expressions are in excellent agreement with the results of the exact solution shown in figures 2 and 3. The comparison is presented in table 1.

It is of interest to point out also that the asymptotic analysis just developed can be extended with ease to the case where $NuT^{-\frac{1}{4}}$ is $O(1)$. Thus, by employing the boundary condition $D_0G = NuT^{-\frac{1}{4}}G$ at $z = 0$, we arrive again at (13) but with $M_0(1+Nu/\alpha_0 T^{\frac{1}{4}})^{-1}$ replacing M_0, so that figure 4, which depicts the right-hand side of (13), can still be used to compute the new critical values

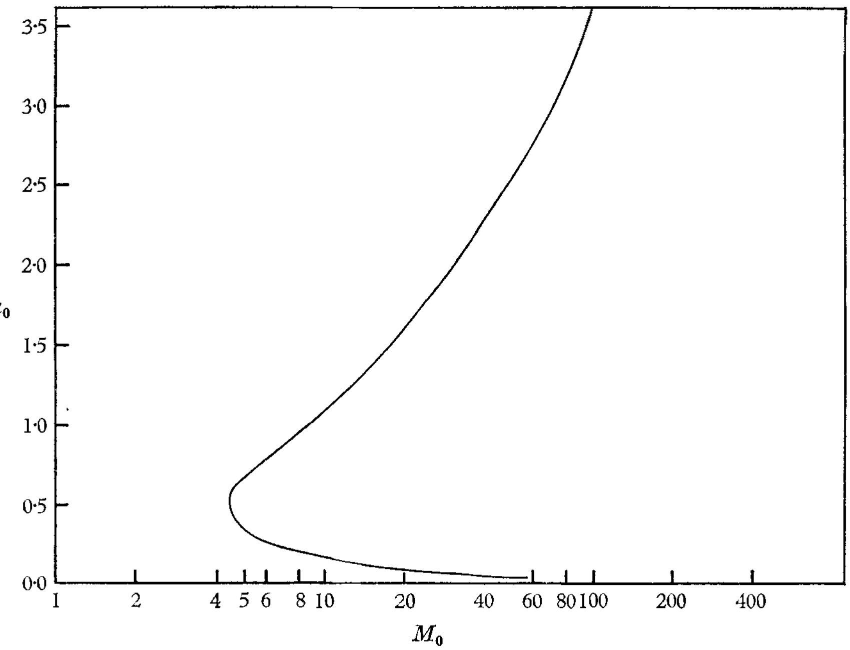

FIGURE 4. M_0 vs. α_0 corresponding to the asymptotic limit of an infinite Taylor number.

Taylor number	Exact solution M_c	Exact solution α_c	Asymptotic solution M_c	Asymptotic solution α_c
0	80	2·0	—	—
10^2	92	2·2	—	—
10^3	164	3·0	140	2·8
10^4	470	5·0	442	5·0
10^5	1400	8·6	1400	8·9
10^6	4420	15·8	4420	15·8

TABLE 1. Comparison between the exact values of M_c and α_c (for the case $Nu = 0$) and those computed from the asymptotic solution (14)

of α_0 and M_0. These become, for example, 0·60 and 12·60, respectively, when $NuT^{-\frac{1}{4}} = 1$.

The solution of the stability problem for surface-tension-driven convection as $T \to \infty$ differs of course in a number of important aspects from the corresponding solution of the buoyancy problem. The latter has been analysed by Chandrasekhar (1961) and more fully by Roberts (1965) and by Niiler & Bisshopp (1965). In particular, Niiler & Bisshopp have shown that as long as the planes $z = 0$ and $z = 1$ remain non-deformable so that $f = 0$, the asymptotic forms for the critical Rayleigh number R_c and the critical wave-number α_c become

$$R_c \to 8{\cdot}69T^{\frac{2}{3}} \quad \text{and} \quad \alpha_c \to 1{\cdot}31T^{\frac{1}{6}} \quad \text{as} \quad T \to \infty. \tag{15}$$

This result indicates, upon comparison with (14), that for large values of T, rotation is somewhat more effective in damping buoyancy-driven convection than surface-tension-driven flow. Another important difference is that, in the buoyancy case, the presence of Ekman layers at $z = 0$ and $z = 1$ can be disregarded to a first approximation when computing asymptotic values for R_c and α_c, since the two expressions in (15) arise from the non-vanishing solution of the stability equations inside the 'core' $0 < z < 1$ which excludes the Ekman layers. Of course, such differences are not surprising since the two problems are quite dissimilar both from the physical as well as from the mathematical point of view, the latter arising primarily from the fact that in the surface-tension case the eigenvalue occurs in one of the boundary conditions rather than in the basic differential equation as in the Rayleigh problem.

In closing this section it is worth pointing out that in our analysis so far we have considered only the case of an isothermal boundary at $z = 1$, having ignored the equally important case of the 'insulating' boundary condition $g'(1) = 0$. We have already remarked, however, that the two asymptotic expressions in (14) apply irrespective of the boundary conditions at $z = 1$, and since, as can be seen from figures 2 and 3, a knowledge of the asymptotic behaviour of the solution as $T \to 0$ (already computed by Pearsons 1958) and as $T \to \infty$ is sufficient to allow us to estimate the full M_c vs. T curve by a simple interpolation, it did not seem worth while to repeat the exact calculations for the insulating case.

4. The nature of the marginal state

Now that the conditions for the onset of stationary convection have been determined it is desirable to examine the possibility that the convection could set in via an oscillatory mode. Actually, since an earlier analysis (Vidal & Acrivos 1966) showed that for $T = 0$ the marginal state is a stationary one, it appears sufficient to consider only the asymptotic case $T \to \infty$. Thus letting $\sigma = i\gamma T^{\frac{1}{3}}$ with γ real, one obtains in place of (11)

$$[(D_0^2 - \alpha_0^2)(D_0^2 - \alpha_0^2 - i\gamma)^2 + D_0^2]f = 0, \quad (D_0^2 - \alpha_0^2 - i\gamma Pr)G = f \tag{16}$$

subject to the conditions

$$f = D_0 G = (D_0^2 - \alpha_0^2)(D_0^2 - \alpha_0^2 - i\gamma)f = 0, \quad D_0^2 f = \alpha_0^2 M_0 G \quad \text{at} \quad z = 0$$

and $G \to 0, f \to 0$ as $z \to \infty$.

The method of solution is quite similar to that presented in the previous section and results in a characteristic equation of the form

$$M_0 = RM(\alpha_0, Pr, \gamma) + iIM(\alpha_0, Pr, \gamma),$$

where RM and IM are real valued functions. However, since M_0 is real this requires that IM be set equal to zero.

The function $IM(\alpha_0, Pr, \gamma)$ was computed numerically for $Pr = 0{\cdot}1$, $0{\cdot}25$, $0{\cdot}5$, $0{\cdot}7$, $1{\cdot}0$ and $7{\cdot}0$ and for $\alpha_0 = 0{\cdot}1, 0{\cdot}5, 1{\cdot}0, 2{\cdot}0, 5{\cdot}0, 10{\cdot}0$, and, although γ was varied from 0 to 10 in steps of 0·2, the only real root of IM was found to occur at $\gamma = 0$. This is of course indicative of a stationary neutral state. Thus, the possibility of overstability has been excluded for the two extreme cases $T = 0$ and $T \rightarrow \infty$, which in turn makes it highly unlikely that it could arise for intermediate values of T. This is in contrast with the analogous problem in buoyancy-driven convection for which, as shown by Chandrasekhar (1961), overstability does indeed manifest itself for $Pr \leqslant 0{\cdot}677$.

5. Experimental observations

Evaporative convection in liquid layers undergoing uniform rotation was studied experimentally by means of a schlieren system already described by Berg *et al.* (1966). The liquid was placed in a circular glass dish 6 in. in diameter which in turn was set on a glass turntable that could be rotated at speeds up to half a revolution per second. The fact that the curvature of the surface increased with increasing angular velocity was found to be a limiting factor in obtaining high values of the Taylor number for very shallow layers, since the speed of rotation was always controlled so as to keep the meniscus height at a value less than about 15 % of the average depth of the liquid pool. In addition, unavoidable vibrations in the system (due to small eccentricities, vibration of the motor, etc.) caused circular waves to propagate over the liquid surface and thus to interfere with the visualization of the flow. These waves were eliminated by introducing a circular beach having a slope of 30° and covering the outer half of the bottom of the dish.

The morphology of the convective motion was studied experimentally using a variety of pure organic liquids and binary mixtures, but the majority of the experiments were carried out with a 50 % solution by volume of ethyl ether in *n*-heptane in which the more volatile component (ether) has a lower surface tension while being denser than *n*-heptane. Thus, upon evaporation of ether, a potentially unstable configuration results with respect to the surface-tension mechanism alone. Of course, temperature variations will also set in which are destablizing both with respect to surface-tension-driven as well as to buoyancy-driven flow, but, for most evaporating liquid mixtures and certainly for the ether–heptane solution in question, these temperature effects are minor compared with those brought about by composition differences between the surface and the bulk. Hence, such systems may be considered isothermal for the purposes of this work.

It has been reported previously (Berg *et al.* 1966), that the convective pattern of evaporating shallow layers (depth less than 0·5 cm) shows a remarkable cellular structure which, upon increase in the depth ($d > 1{\cdot}0$ cm), becomes highly irregular and non-stationary. Now, it would be expected from the mathematical analysis that, when a moderately deep layer of an evaporating liquid is rotated, the stabilizing effect of the Coriolis force would bring the system closer to its marginal state and that above a certain value of the Taylor number a cellular convective pattern would be recovered. This change from a random motion to a regular flow structure with increasing T was observed in all the liquids that were studied. The steady pattern corresponding to a value of $T \geqslant 10^3$ was always found to be one of hexagonal cells which, once established, diminished in size as the Taylor number was further increased.

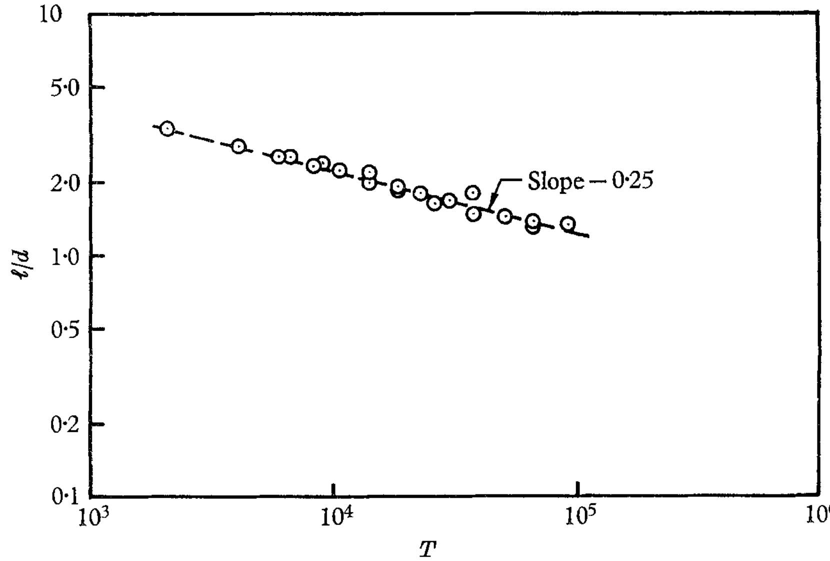

FIGURE 6. Decrease of cellular diameter–depth ratio, ℓ/d, with increasing Taylor number T for a 50 % solution of ether in n-heptane.

The effect of the Taylor number T on the convective pattern of an evaporating solution of ether in n-heptane is illustrated in figure 5 (plate 1), and the variation of the cell diameter ℓ with T in figure 6. It is seen from figure 6 that, for $T > 10^4$

$$\ell/d \approx 12T^{-\frac{1}{4}} \quad \text{or} \quad \alpha = 0{\cdot}7T^{\frac{1}{4}},$$

a result that compares very favourably with (14) even though the latter represents the asymptotic solution of the linearized equations describing the marginal state. This is analogous to the situation encountered by Nakagawa & Frenzen (1955) who found that the cell sizes in rotating fluids undergoing buoyancy-driven convection were in very close agreement with the values predicted theoretically. This agreement is all the more remarkable in the present case considering the fact that the stabilizing effect exerted by the 'favourable' density gradient which is produced by the rapid depletion of ether from the surface has not been taken into account in the mathematical analysis.

Another aspect of the experimental studies consisted of examining how far the convective motion propagated below the liquid surface. Glass microballoons were suspended in the fluid and their motions were followed visually as they were convected by the moving liquid. It was found that when the evaporating pool

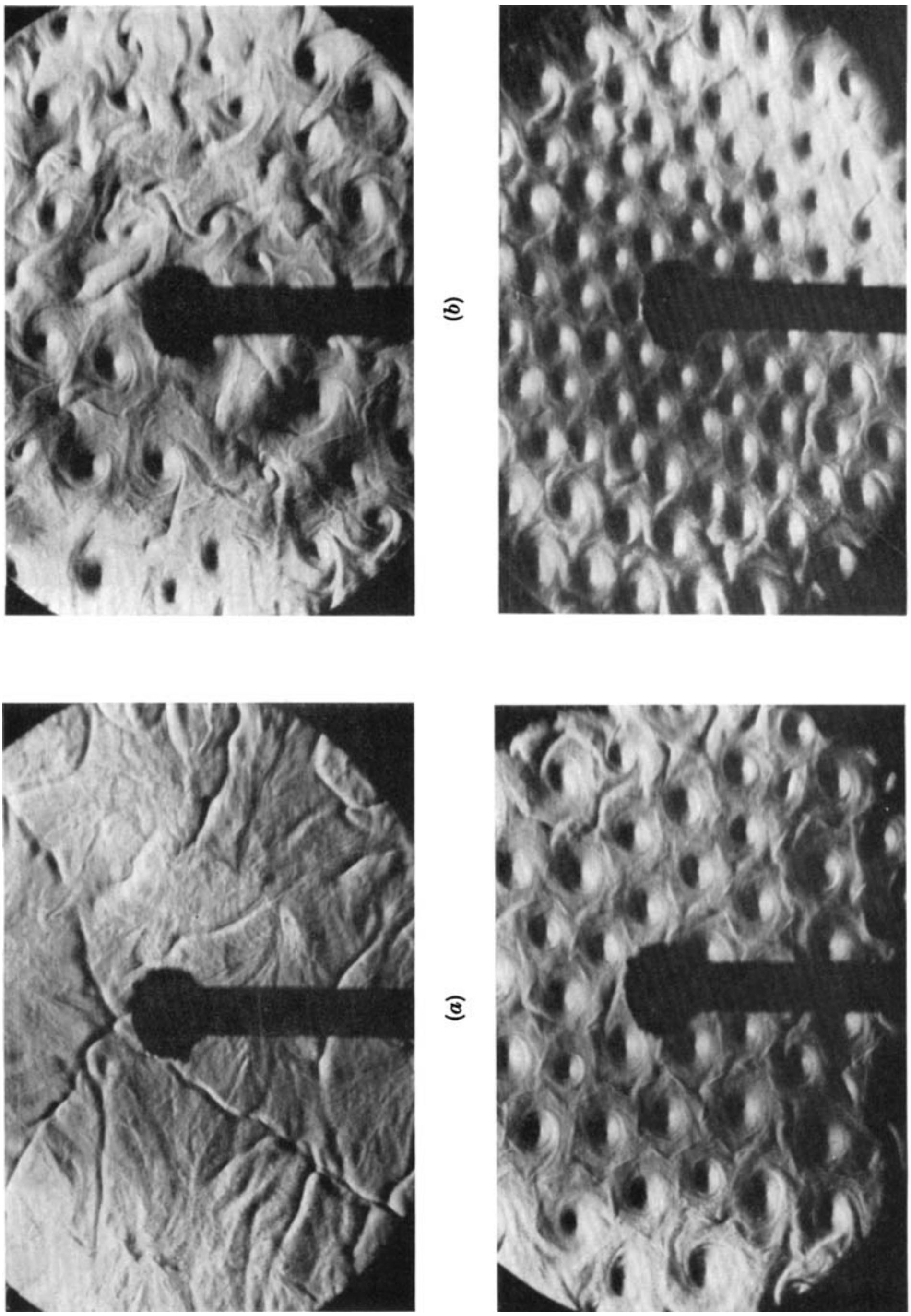

FIGURE 5. Changes of the convective pattern with increasing Taylor number in a 0·5 cm deep pool of a solution of ethyl ether in *n*-heptane. (*a*) $T = 0$; (*b*) $T = 5600$; (*c*) $T = 18{,}700$; (*d*) $T = 66{,}000$.

was not rotating the convective currents penetrated down to the bottom of the layer, but that for $T > 10^3$ the three-dimensional motions were confined to a small portion of the liquid adjacent to the free surface with the bulk of the fluid moving along closed streamlines characteristic of solid-body rotation. Moreover, it was observed that the convective sublayer became smaller as the speed of rotation was increased. This is, once again, in qualitative agreement with the results of the asymptotic analysis for large T according to which the velocity and temperature or concentration fluctuations at the marginal state should be confined to a layer near the surface of thickness proportional to $T^{-\frac{1}{4}}$.

At this point it is of interest to note that according to the asymptotic analysis for large T the velocity fluctuations near the marginal state are greater than the dimensionless temperature or concentration fluctuations by a factor of $T^{\frac{1}{2}}$. In view of the previous remarks regarding the agreement between theory and experiment it might also be expected then that this property of the system would be carried over to the final state characterized by finite amplitudes. Experimentally no appreciable variations of the convective velocity were observed as the Taylor number was changed over a wide range of values ($10^3 \rightarrow 10^6$) and it is reasonable to suppose therefore that the dimensionless temperature or concentration fluctuations became smaller as T was increased.

The sense of the circulation in the convective cells was also determined experimentally by sprinkling small amounts of lycopodium powder on the surface of the liquid. The motion of the particles was then followed visually. When a 50 % solution of ether in heptane was allowed to evaporate it was observed that the tracer particles were swept away from the cell centres and that they tended to accumulate at the periphery, thus indicating that the liquid was rising in the core of the cell and descending along the cell boundaries. Similar experiments were repeated using a liquid for which only the buoyancy mechanism was operative, namely water (Berg *et al.* 1966), where it was found that the tracers now moved away from the cell periphery and accumulated at the cell centre. This was in agreement with the earlier observations of Nakagawa & Frenzen (1955).

6. Conclusions

The main conclusions that can be drawn from the present work on rotating liquid layers for which convection is driven by surface-tension gradients are the following:

Uniform rotation enhances the stability of the system. Both the critical Marangoni number M_c and the critical wave-number α_c are monotonically increasing functions of the Taylor number T, having asymptotic forms $M_c = 4{\cdot}42T^{\frac{1}{2}}$ and $\alpha_c = 0{\cdot}5T^{\frac{1}{4}}$, respectively, as $T \rightarrow \infty$. This asymptotic result indicates that at the marginal state the characteristic cell dimension ℓ does not depend on the depth d of the layer and that the critical vertical temperature gradient β is independent of d and, surprisingly, of ν, the kinematic viscosity of the fluid.

The marginal state of the system is stationary.

In the limit $T \to \infty$ the temperature and velocity fluctuations are confined to an Ekman layer whose thickness is proportional to $T^{-\frac{1}{4}}$.

For large values of T the dominant convective pattern consists of hexagonal cells that diminish in size as T increases, in good agreement with the results of the linearized theory.

The prediction based on the linearized analysis that, as $T \to \infty$, the velocity fluctuations should be $O(T^{\frac{1}{2}})$ larger than the temperature fluctuations relative to the mean temperature drop, and the observed fact that the former are independent of T seem to indicate that whenever the convection is driven by surface tension these normalized temperature fluctuations across the surface of an evaporative liquid layer should become smaller as the speed of rotation is increased.

In the presence of uniform rotation, the direction of the flow is upward in the core and downward along the cell boundaries when the convection is surface-tension-driven, whereas the reverse holds whenever the convection is driven by buoyancy and the system is cooled from above.

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